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ANALYSIS OF PILES USED FOR SLOPE STABILIZATION

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SUMMARY

Piles used for the stabilization of slopes have to be adequately designed to resist the induced lateral loads due to the movement of the unstable slope. In this paper, a numerical method is presented for the analysis of this problem. In this approach, the piles are modelled using beam finite elements. The soil response at the individual piles is modelled using the modulus of subgrade reaction and pile–soil–pile interaction considered using the theory of elasticity. Two case histories, one for single pile and the other for pile group, are analysed which show that the numerical model can predict the general characteristics of the piles reasonably well. The study suggests that the design of the piles based on the computed response from single pile analysis, ignoring group effects, may be unduly conservative.

KEY WORDS: slope stabilization; piles; soil response modelling

INTRODUCTION

Piles have been used for the stabilization of slopes for many decades. One of the main design issues is the determination of the shear forces and bending moments developed in the piles caused by the movement of the unstable slope. Although a number of successful application of this method of slope stabilization have been reported in the literature (e.g. References 1–3), there is generally a lack of a widely accepted method of analysis and design of these piles.

Ito and Matsui[4] presented a theoretical method of estimating the lateral forces acting on a row of piles by considering plastic flow of the soil through the piles; the characteristics of the piles are, however, not considered in the analysis. Viggiani[5] postulated various failure mechanisms of a rigid pile under the action of the sliding soil. Hence, the method is only applicable for piles at failure, and the influence of group effects is not considered.

In this problem, the soil movements caused by the slope instability induce lateral forces on the stabilizing piles which interact with one another. Therefore, a consistent and rational method of analysis should consider all elements of this soil–pile interaction problem.

A method of analysis based on a simplified boundary element method has been reported by Poulos[6,7] and Lee *et al.*[8] for single piles. The approach is based on the use of Mindlin's solution to model the soil response and the influence of non-homogeneous soil is treated in an approximate manner.

This paper presents a numerical model where the piles are modelled using beam finite elements and the soil is modelled using a hybrid method of analysis which simulates the soil response at individual piles using the subgrade reaction modulus and pile–soil–pile interaction using the

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theory of elasticity. This method of analysis is considerably simpler than the simplified boundary element method described above. The ability of this method to predict the behaviour of piles subject to lateral soil movements due to slope instability is verified through retrospective analysis of well-documented case histories.

METHOD OF ANALYSIS

The problem considered is a group of vertical piles installed in one or more rows to stabilize a slope (see Figure 1). The piles are assumed to be linear elastic with diameter d. The behaviour of the pile is characterized by the Young's modulus E_p, the second moment area I_p and pile length L. The soil profile may be non-homogeneous. The soil movements of the unstable soil above the sliding surface induce lateral forces on the piles which interact with one another.

The problem can be analysed by considering the pile–soil interaction forces acting on the piles and the soil separately and then combining these two by the consideration of equilibrium and compatibility. In this method of analysis, the individual piles are modelled using beam finite elements. The soil response at the individual piles is modelled using the modulus of subgrade reaction and pile–soil–pile interaction is considered using the theory of elasticity. This approach is an extension of an earlier method used for the analysis of pile groups subject to applied loads.[9–11].

The load–deflection relationship of the group piles can be written as

$$[K_p]\{y_p\} = \{P_p\} \tag{1}$$

where $[K_p]$ is the assembled stiffness matrix of all the beam elements forming the piles, $\{y_p\}$ is the vector of pile deflections and rotations of the pile nodes, and $\{P_p\}$ is the vector of pile–soil interaction forces acting on the pile. The stiffness matrix of the beam elements is available in many text (e.g. Reference 12). The interaction forces vector $\{P_p\}$ considers only the horizontal interaction forces; the corresponding components for the moments are zero.

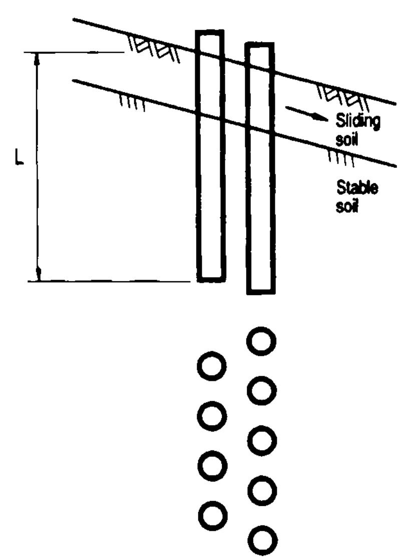

Figure 1. Piles in unstable slope

The soil response at the individual piles is modelled using the modulus of subgrade reaction. The lateral soil pressure (p_s) acting on the pile is given by

$$p_s = k_h(y_s - y_0) \tag{2}$$

where k_h is the modulus of subgrade reaction of the soil, y_s is the soil deformation at the pile–soil interface, and y_0 is the lateral soil movement due to slope instability. The lateral force of the soil (P_s) acting on the pile at a particular node is

$$P_s = k_h dl(y_s - y_0) \tag{3}$$

or

$$P_s = K_h l(y_s - y_0) \tag{4}$$

where l is the pile element length associated with that node and $K_h(=k_h d)$ is the soil stiffness per unit length of the pile.

The relative lateral displacement of the soil at the pile–soil interface at a particular node i of a pile due to interaction forces acting on itself and at other nodes in the group piles can be obtained by superposition from

$$y_{si} - y_{0i} = \sum_{j=1}^{j=N} f_{ij} P_{sj} \tag{5}$$

where y_{si} is the lateral soil deformation at the pile–soil interface at node i, y_{0i} is the lateral soil movement at node i due to the unstable slope in the absence of the piles, f_{ij} is the flexibility coefficient denoting the lateral deformation of the soil at node i due to a unit pile–soil interaction lateral force acting at node j, P_{sj} is the pile–soil interaction lateral force acting at node j, and N is the total number of nodes. The flexibility coefficients are determined as follows:

(a) For $i = j$, $f_{ii} = 1/(K_h l)$ which is the reciprocal of the soil spring stiffness at the node.
(b) For nodes j associated with the same pile as node i and $i \neq j$, $f_{ij} = 0$ indicating no interaction between nodes of the same pile; this is the inherent assumption of the subgrade reaction modulus method for modelling soil behaviour of individual piles.
(c) For interaction between nodes of different piles, i.e. pile–soil–pile interaction, the flexibility coefficients are determined using Mindlin's equation for the influence of a unit horizontal point force in a homogeneous isotropic elastic half-space. The flexibility coefficients for a non-homogeneous soil profile are approximated using the average of the soil Young's modulus at nodes i and j.

Equation (5) is written for each of the nodes leading to the following flexibility relationship of the soil:

$$\{y_s\} - \{y_0\} = [F_s]\{P_s\} \tag{6}$$

where $\{y_s\}$ is the vector of soil deformations at the pile nodes, $\{y_0\}$ is the vector of lateral soil movements at the pile nodes in the absence of the piles, $[F_s]$ is the soil flexibility matrix, and $\{P_s\}$ is the vector of pile–soil interaction forces acting on the soil. The vectors $\{y_s\}$, $\{y_0\}$ and $\{P_s\}$ are augmented with zeros in the appropriate rows to correspond to the rotation of the pile nodes to be compatible with the vector $\{y_p\}$ in equation (1). The soil flexibility matrix $[F_s]$ is augmented with zeros in the appropriate rows and columns corresponding to the rotational degrees of freedom of the pile to be compatible with the assembled stiffness matrix of the pile elements $[K_p]$ in (1); the corresponding main diagonal terms are assigned a 'large value' to represent the small rotational stiffness of the soil.

The flexibility relationship in (6) is inverted to give the following stiffness relationship of the soil:

$$\{P_s\} = [F_s]^{-1}(\{y_s\} - \{y_0\}) = [K_s](\{y_s\} - \{y_0\}) \tag{7}$$

where $[K_s]$ is the stiffness matrix of the soil.

Equilibrium of the interaction forces at the pile–soil interface yields

$$\{P_s\} = -\{P_p\} \tag{8}$$

and compatibility of the deformations of the pile and the soil (assuming linearity) yields

$$\{y_s\} = \{y_p\} \tag{9}$$

Using the above equilibrium and compatibility conditions together with (1) and (7) yields the following stiffness relationship of the pile group system:

$$([K_p] + [K_s])\{y_p\} = [K_s]\{y_0\} \tag{10}$$

The vector $[K_s]\{y_0\}$ represents the induced lateral forces acting on the piles resulting from the lateral soil movements of the unstable slope. This set of equations can be solved for the lateral soil movements caused by the slope instability to give the pile deflections and rotations from which the shear forces and bending moments can be obtained.

The non-linear behaviour of the soil can be incorporated by limiting the soil pressure that can act on the pile using an iterative technique as described by Chow[13] for axially and laterally loaded piles.

SOIL PARAMETERS

The soil parameters required for the analysis are: the modulus of subgrade reaction k_h (or the lateral soil stiffness per unit length of the pile $K_h = k_h d$), the limit soil pressure p_y, the Young's modulus (E_s) and Poisson's ratio (ν_s) of the soil.

Lateral soil stiffness and Young's modulus

The lateral soil stiffness per unit length of the pile is assumed to be related to the Young's modulus of the soil[14] as follows:

$$K_h \simeq E_s \tag{11}$$

For clays, the Young's modulus is usually correlated to the undrained shear strength c_u as follows:

$$E_s = \beta_1 c_u \tag{12}$$

where β_1 typically lies between 150 and 300.[14]

For sands, the Young's modulus can be correlated to the SPT blow count (N) as follows:

$$E_s = \beta_2 N \quad (\text{in MN/m}^2) \tag{13}$$

where β_2 typically lies in the range of 1–1·5.

Limit soil pressure

For piles in cohesive soils, the limit yield pressure due to the relative pile–soil displacement is given by

$$p_y = N_p c_u \tag{14}$$

where N_p is limit yield pressure coefficient. From the work of various researchers summarized by Viggiani,[5] the following values for N_p can be used. For the sliding soil above the slip surface, $N_p = 3$–4, and for the stable soil below the slip surface, $N_p = 8$–12. The lower value of N_p in the sliding zone is due to its proximity to the ground surface and the weakening of the soil due to the sliding movements.

For piles in cohesionless soils, the limit yield pressure may be determined based on the work of Broms[15] as follows:

$$p_y = 3K_p\sigma'_v \tag{15}$$

where K_p is the Rankine passive pressure coefficient, and σ'_v is the effective overburden pressure.

The limit soil pressures given in equations (14) and (15) are strictly for single piles. In the case of group piles, the limit soil pressures may be affected by the pile spacing and group arrangement. However, these limit values for group piles are not available in the literature. Hence, until such values are available, the limit pressures given in equations (14) and (15) may also be used for group piles.

ANALYSIS OF CASE HISTORIES

Single pile in a sliding slope

A field test in which is reinforced concrete pile was installed in a sliding slope consisting mainly of clay was reported by Esu and D'Elia.[16] Lateral movements of the slope took place in the upper 7·5 m layer. The pile was instrumented to determine the shear forces, bending moments, pile deflections and rotations. The test pile was 0·79 m in diameter with a length of 30 m. The bending stiffness of the pile (E_pI_p) was 360 MN m^2.

Poulos[7] analysing this problem using a simplified boundary element method quoted the work of Maugeri and Motta[17] which suggested that the undrained shear strength c_u might be 40 kN/m^2 and the values of the lateral limit yield pressure coefficient of the soil N_p could be 3 and 8 for the upper shallow moving soil layer and the lower stable soil layer, respectively. To match the measured bending moment profile, Poulos made the following two assumptions regarding the soil in his analysis:

(i) the soil Young's modulus E_s increases linearly from zero at the surface to 16 MPa at the pile toe; and
(ii) since the soil movement profile to cause the pile to yield was not reported, a uniform distribution of lateral soil displacement of 110 mm from the ground surface down to the sliding surface 7·5 m below the ground surface was used.

It appears that assumption (i) was made to match the computed bending moment profile of the pile with the measured profile. However, in practice, it would be difficult to determine this soil modulus distribution *a priori* in design. Hence, this problem was analysed using the present approach with the usual way of estimating the soil parameters for clay as discussed earlier. From equations (11) and (12), the Young's modulus of the clay was assumed to be $E_s = 200c_u$ and the

soil stiffness per unit length of the pile $K_h = E_s$. The sliding soil movement profile was taken to be the same as Poulos.

The bending moment, shear force, pile deflection and pile rotation profiles predicted using the present approach are compared with the field measurements and the results of Poulos in Figures 2, 3, 4 and 5, respectively. The results of the present method generally compare favourably with the field measurements. The results of Poulos are, as expected, closer to the field measurements since the soil Young's modulus distribution was chosen to match the measured bending moment profile.

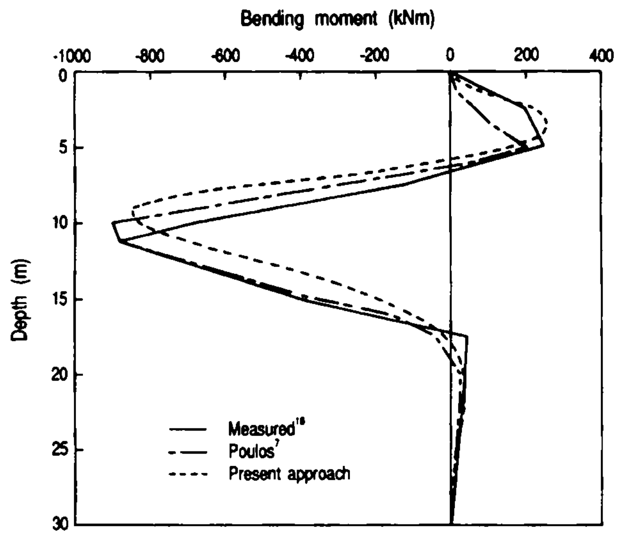

Figure 2. Test of Esu and D'Elia:[16] Bending moment profile

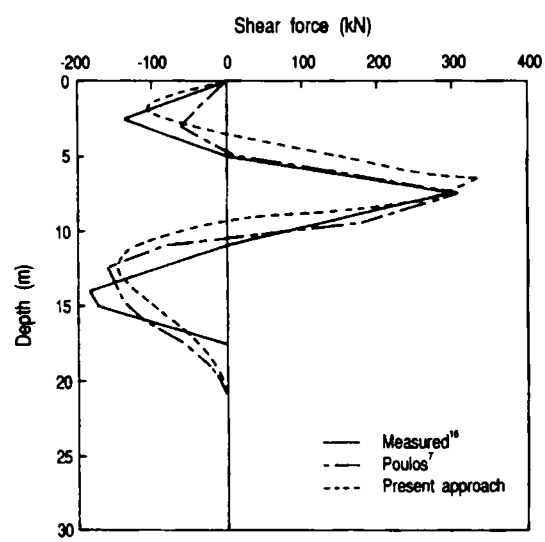

Figure 3. Test of Esu and D'Elia:[16] Shear force profile

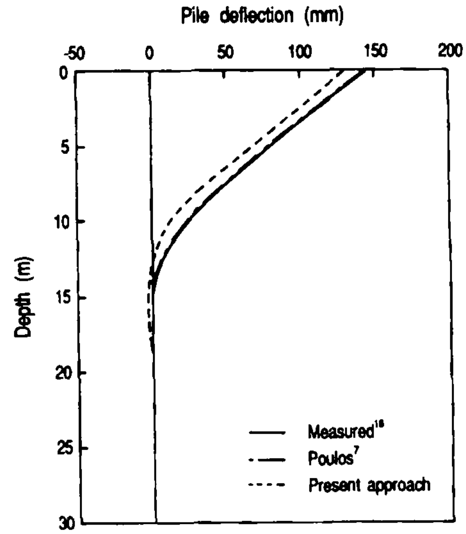

Figure 4. Test of Esu and D'Elia:[16] Pile deflection profile

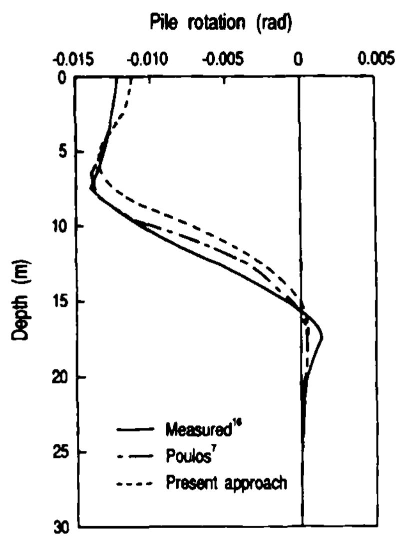

Figure 5. Test of Esu and D'Elia:[16] Pile rotation profile

Group piles used to stabilize landslide

At the 36th kilometer, near Aktea, of the national road from Athens to Sounion in Greece, slope instability problems have caused cracks to appear in the road pavement of a semi-bridge structure, 2 yr after construction; a rather large inclination of the wing wall of the bridge was also observed.[18] The site consists of neogene lacustrine deposits which include alternating layers of

conglomerates and clayey or sandy marls with lenses of sandstones. The sliding surface was about 4 m from the ground surface.

The solution used to stabilize the landslide at the area of the semi-bridge was by means of two rows of 1 m diameter concrete bored piles of 12 m length with a distance of 2.5 m between them.[8] The piles were installed in the vicinity of the foundation piers of the semi-bridge atthe downhill area as shown in Figure 6. Two of these piles were replaced by steel pipe piles with an external diameter of 1 m, wall thickness of 18 mm and flexural stiffness ($E_p I_p$) of 1540 MN m^2 which was about the same as that of the concrete bored piles. These steel piles were instrumented with vibrating wire strain gauges and inclinometers.

The lateral soil movements shown in Figure 7 were recorded using inclinometer G13,[18] located uphill of the experimental site (see Figure 6). These measurements indicate that the sliding surface

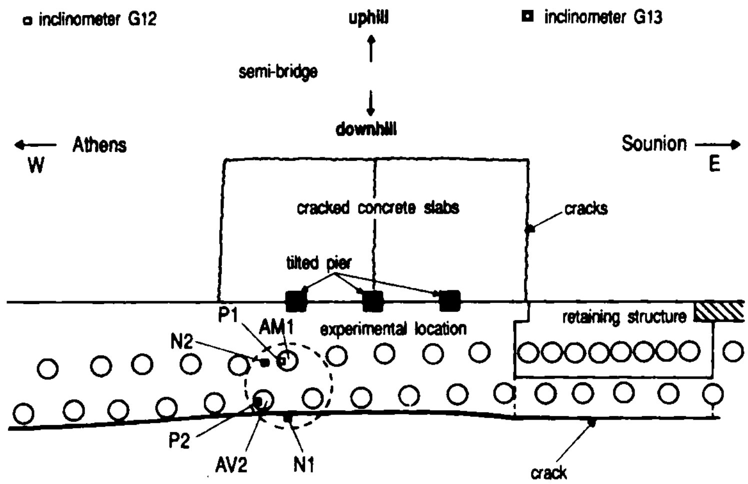

Figure 6. Plan view of the location of the stabilizing piles and instrumented piles[18]

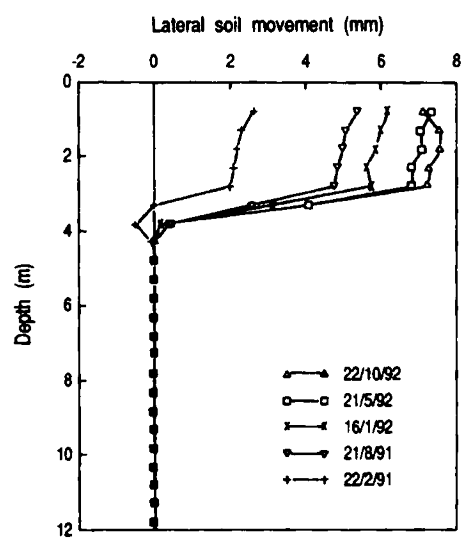

Figure 7. Measured lateral soil movement profiles[18]

was about 4 m below the ground surface and these values were taken to be the soil movements in the absence of the piles in the analysis. From the pressuremeter tests, the following average parameters were obtained. For the soil above the slip surface: $p_l = 0{\cdot}9$ MPa and $E_M = 15$ MPa, and for the soil below the slip surface: $p_l = 3{\cdot}2$ MPa and $E_M = 70$ MPa, where p_l is the pressuremeter limit pressure and E_M is the pressuremeter modulus.

In the analysis, it was assumed that the soil stiffness $K_h = E_M$ and Young's modulus of the soil $E_s = E_M$. The Poisson's ratio of the soil $\nu_s = 0{\cdot}5$. The undrained shear strength of the soil can be estimated from the pressuremeter test results according to the following relationship given by Baguelin *et al.*:[19]

$$c_u = (p_l - p_0)/\beta \tag{16}$$

where p_0 is the total horizontal pressure and β is a coefficient which, from test results, lies between 6.5 and 12, with an average of 9. Observations from stand pipes showed no evidence of water above the sea level. Since it is difficult to estimate the at-rest coefficient of lateral earth pressure, K_0, and the values of the measured limit pressures are high, the undrained shear strength can be estimated approximately from

$$c_u \simeq \frac{p_l}{9} \tag{17}$$

Hence, the undrained shear strength of the soil above the slip surface is $c_u = 100$ kPa, and below the slip surface, $c_u = 356$ kPa. The lateral limit yield pressure coefficients N_p (see equation (14)) were taken to be 3 and 8 for the soil mass above, and below the slip surface, respectively.

The problem was first analysed to study the effect of pile–soil–pile interaction using the lateral soil movement profile measured on 22 October 1992 shown in Figure 7. With the instrumented pile at the centre, the influence of the adjacent piles was studied by considering an increasing number of piles in the group within a certain radius of influence from the centre pile. This was studied by considering a single pile, 5 piles, 9 piles, 13 piles and 17 piles which correspond to a radius of influence of 0d, 2·5d, 5d, 7·5d and 10d, respectively (see Figure 6). It can be seen in Figure 8 that the bending moments and pile deflections decrease with increase in the number of

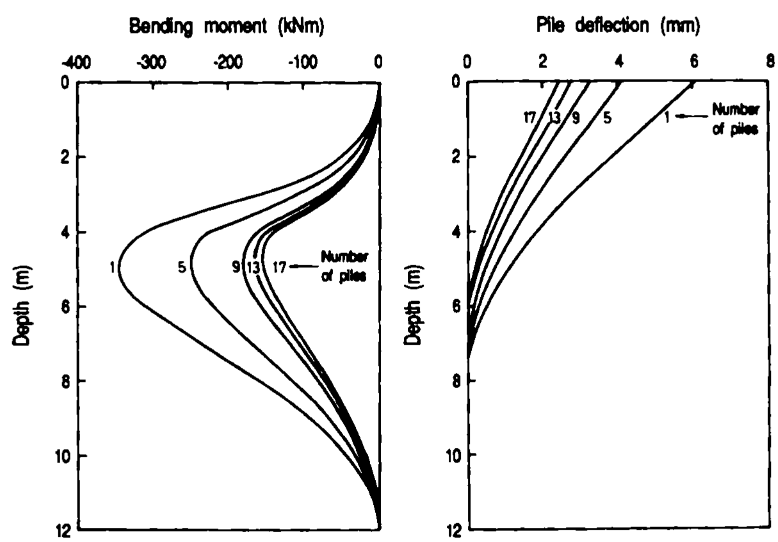

Figure 8. Effect of pile–soil–pile interaction on pile behaviour

piles considered in the group which shows that pile–soil–pile interaction has the beneficial effect of reducing the bending moments and pile deflections. The rate of decrease of these parameters reduces with increasing radius of influence. For practical problems, a radius of influence of about 10 pile diameters (or 10 m in this case) may be considered satisfactory to optimize computational resources. Moreover, the use of the theory of elasticity for pile–soil–pile interaction has been

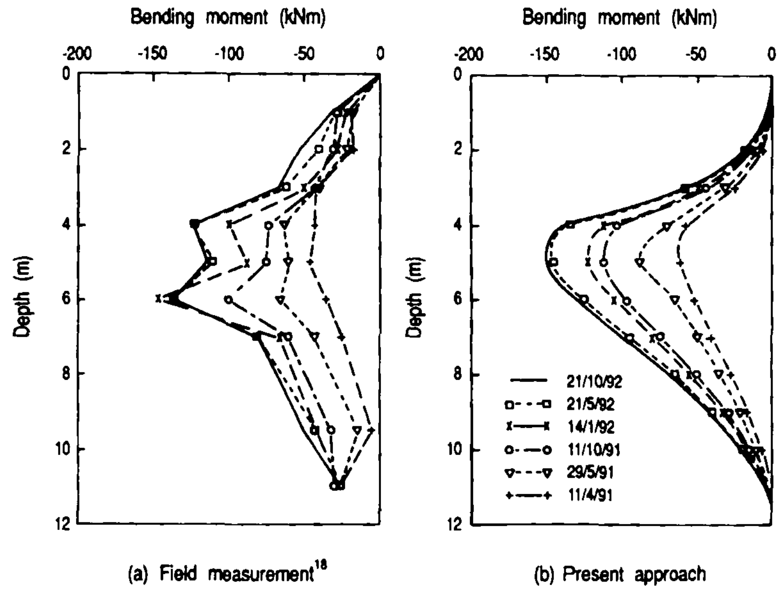

Figure 9. Test of Kalteziotis *et al.*:[18] Bending moment profiles

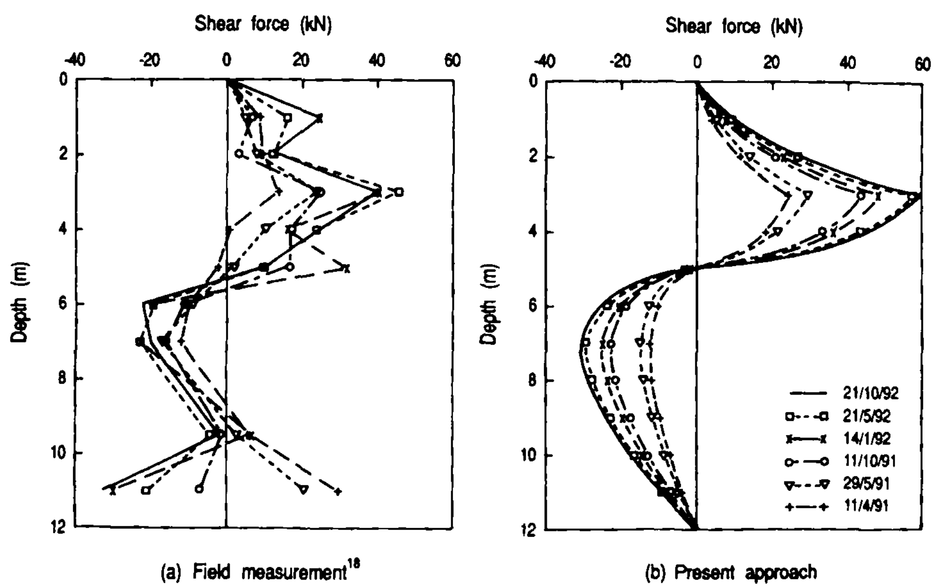

Figure 10. Test of Kalteziotis *et al.*:[18] Shear force profiles

known to overestimate interaction effects. Hence, the subsequent results presented have been based on 10 pile diameters (or 10 m) radius of influence, i.e. considering 17 piles.

The computed bending moments, shear forces, pile deflections and pile rotations of the instrumented pile are shown in Figures 9, 10, 11 and 12, respectively. The computed responses of the pile on the days indicated in the figures were obtained using the measured soil movement

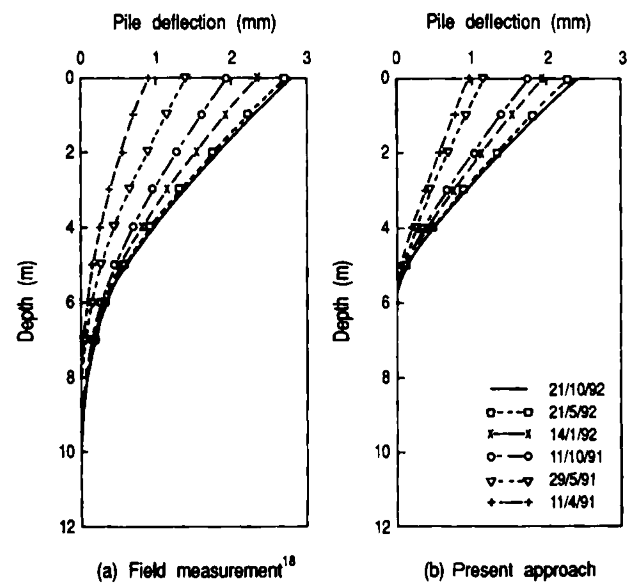

Figure 11. Test of Kalteziotis *et al.*:[18] Pile deflection profiles

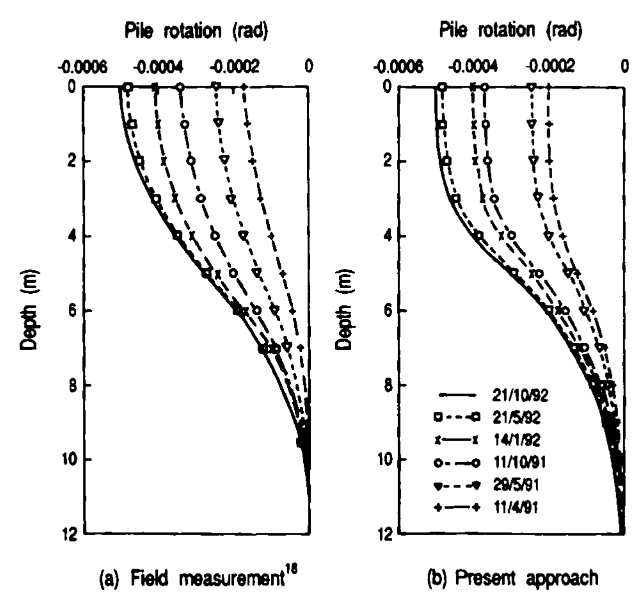

Figure 12. Test of Kalteziotis *et al.*:[18] Pile rotation profiles

profiles shown in Figure 7 and interpolating from this figure where necessary. In general, the computed responses of the pile compare favourably with the field measurements[18] in terms of the magnitude and distribution. It is worth pointing out that field measurements are not infallible and they are subjected to several sources of errors. In this instance it can be seen that the measured bending moment and shear force profiles are not smooth and show some oscillations, and the measured shear forces at the pile toe show some erratic behaviour.

CONCLUSIONS

A numerical method for analyzing piles used for the stabilization of slopes was reported. The approach considers the lateral soil movements of the unstable slope which induce lateral forces on the stabilizing piles.

The performance of piles used for the stabilization of slopes was examined through the retrospective analysis of two well-documented case histories: one for single pile and the other for group piles. The numerical results suggest that the approach is capable of predicting the behaviour of the piles, viz., the magnitude and distribution of the bending moments, shear forces, pile deflections and pile rotations. Pile–soil–pile interaction in group piles was shown to have a significant influence on the performance of the piles. The results of analysis from single pile ignoring group effects are overly conservative, particularly for large pile groups.

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